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The Physics of a Space Elevator

Trevor Hamer
University of Southern Maine

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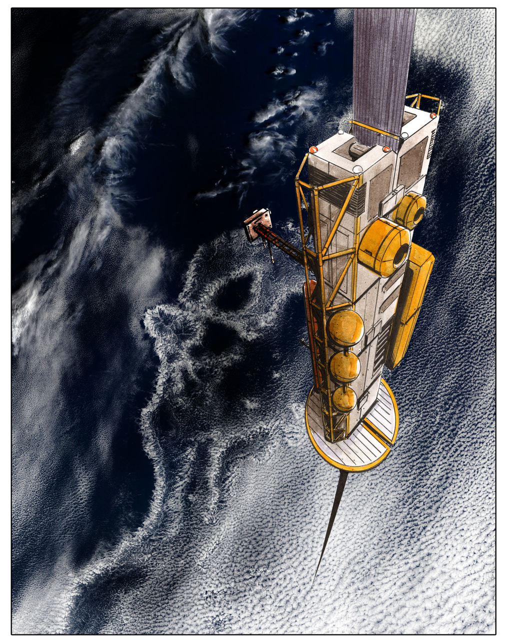
Trevor Hamer and Paul A. Nakroshis

Department of Physics, University of Southern Maine

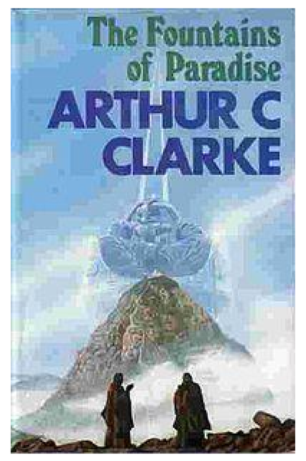
Abstract

A space elevator is a hypothetical device consisting of a long cable attached to the surface of the earth that extends upward into space. Its purpose is to provide a tether on which a vehicle could be lifted up into orbit, greatly reducing the cost of space travel. This project explains the physical forces acting on the elevator along with the kinds of materials required to keep such a cable intact. It also examines different design aspects, as well as potential problems facing the construction and usage of the elevator, and whether or not it is something we should expect to see in the future.



History

The wonderous concept of building a tower to the heavens has for millenia been one of pure fantasy. In 1895, Konstantin Tsiolkovsky, commonly known today as "the father of rocketry," brought the fantasy down to Earth. While looking upon the newly constructed Eiffel Tower, he imagined a similar structure that reached up into space and held a structure at the top: he called it the "celestial castle."



This idea remained relatively unknown until 1979, when science fiction writer Arthur C. Clarke touched upon it in his novel *Fountains of Paradise*. The story, about the engineer of an elevator into space, made the dream seem within grasp. However, an essential part Clarke's elevator was the "hyperfilament," a material with tensile strength far beyond that of any other, which was required to keep the lift cable intact. The idea would remain science fiction until the "hyperfilament" was a reality.

The proposed solution came in the 1990s, when research began on an exotic type of matter: the carbon nanotube. This material's high tensile strength and low density made it a perfect candidate for a space elevator cable. Unfortunately, the production of nanotubes has proven somewhat difficult, with the longest one so far being about 7 inches. Despite this, research, prototype competitions, and campaigns have all been underway with overwhelming support, showing that people have started to take this idea seriously.

A Free-Standing Tower

We can approximate the space elevator cable as a long, free-standing tower attached to the surface of the Earth. Any given length of the tower dr is affected by three forces: the downward gravitational pull of the Earth, the weight of the tower above it, and the normal force of the tower below it, all shown in Fig.1. The acceleration, to complete the equation $\Sigma\vec{F} = m\vec{a}$, is the radial acceleration $\omega^2 r$, as we assume the tower is rotating along with the Earth.

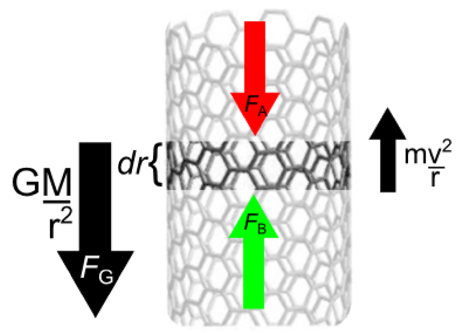


Figure 1: The forces at work on any given section of the elevator cable add up to its radial acceleration, the "centrifugal force."

The forces with their directions, then, give the equation:

$$\frac{GMm}{r^2} - m\omega^2 r = T \qquad (1)$$

where T represents the tension on the segment, or the sum of the forces F_A and F_B.

We can make Eq.1 into something more with some substitutions. The mass of the segment can be replaced with the value $\rho A dr$, where ρ represents the material's density and Adr its volume. The tension can be changed to a tensile stress per unit area dT times its area A. The ω^2 can be rewritten in terms of the geosynchronous orbit radius: $\frac{GM}{R_G^3}$; this is obtained from the fact that $\frac{GM}{r^2} = \omega^2 r$ at this particular radius (see Fig. 2). The equation now becomes:

$$\frac{GM(\rho A dr)}{r^2} - (\rho A dr)\left(\frac{GM}{R_G^3}\right) r = AdT$$

$$= GM\rho\left[\frac{1}{r^2} - \frac{r}{R_G^3}\right] dr = dT$$

which we can integrate from $r = R_E$ (at which $T = 0$ by virtue that the tower is free-standing and not held up) to find the tension as a function of a height above the Earth, r:

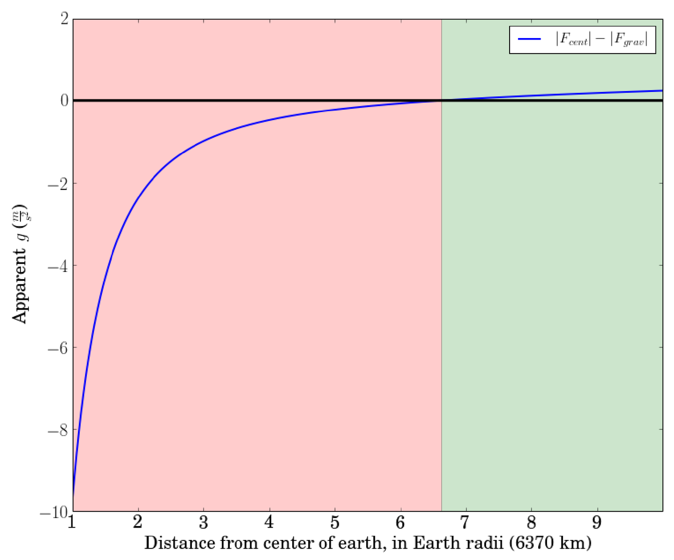


Figure 2: The sum of the downward gravitational force and upward centrifugal force. At the geosynchronous radius, an object feels no inward or outward pull.

$$T(r) = GM\rho\left[-\frac{1}{r} - \frac{r^2 - R_E^2}{2R_G^3} + \frac{1}{R_E}\right] \qquad (2)$$

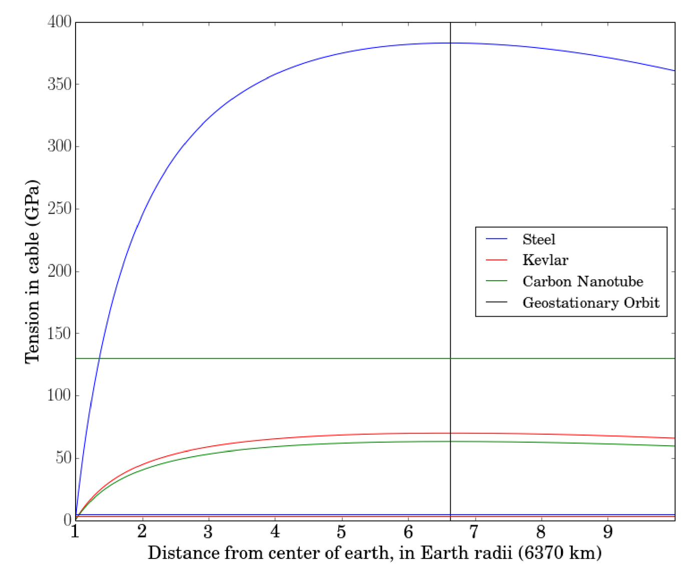


Figure 3: The tension function of Eq. 2 for steel, Kevlar, and carbon nanotube, along with their maximum tensile strength.

With Eq. 2, we can find out that the maximum tension occurs at geosynchronous orbit. Thus, the cable must be constructed with a material that could comfortably endure the tension at this point. From Fig. 3, we can see that two of our strongest building materials fail miserably given their low tensile strength. However, the tension in a cable made out of carbon nanotubes would not even reach half of its maximum stress capacity.

The free-standing model can, of course, be improved further. Rather than having a cable at varying tension, we could create a "tapered" cable with a constant tension throughout - but varying cross-sectional area. While the results of this version still warrant the need for carbon nanotubes, this design allows much more room for additional tension, such as that brought on by a lifter.

Carbon Nanotubes

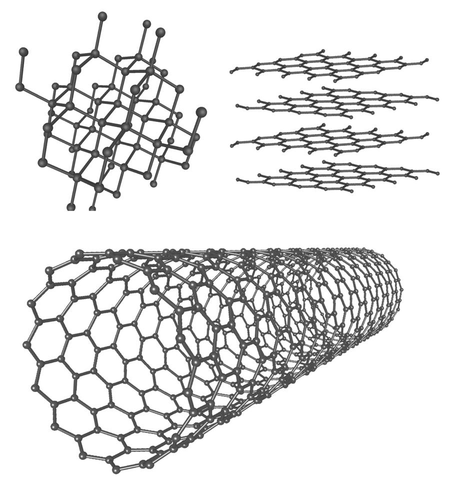


Figure 4: Three allotropes of carbon: diamond (top-left), graphite (top-right), and carbon nanotube (bottom).

In nature, cystalline carbon tends to appear in two forms: diamond and graphite. Diamond is often cited as the hardest mineral on Earth. On the other hand, graphite can be easily ground up by a pencil sharpener. The vast difference in strength is due to chemical bonding. Each atom of carbon in diamond is held together in a tetrahedral formation (called sp^3 hybridization) while graphite is made up of layers of one-atom thick sheets of carbon held together by the weak van der Waals force.

However, if we take a single sheet composing graphite, called graphene, we can see that the carbon atoms are held together by double bonds in a trigonal-planar (sp^2) formation. These bonds are actually even stronger than those in diamond. Carbon nanotubes are essentially "rolled up" sheets of graphene, extending their remarkable chemical, tensile, and conductive properties while being much more useful for engineering purposes.

Problems, Hazards, and Solutions

There are many valid reasons as to why people remain skeptical of the space elevator. Besides the fact that currently-produced carbon nanotubes are not nearly long enough to serve as an elevator cable, a single structural defect within a tube can reduce its tensile strength by 30%. However, research on other varieties of nanotube - such as those made up of silicon or boron nitride - may present us with an alternative in the future.

The durability of the cable is another thing to consider. The cable, only about 5 cm in diameter, would have to endure through its length of about 100,000 km. With all that carbon nanotubes have to offer, they would also have to stand up to acid rain, atmospheric conditions, and extreme space weather. Space junk is another issue - some have suggested a ribbonlike cable to make any collisions less severe.

Getting a vehicle up the cable is a problem in its own. NASA and others have made competitions for vehicles that can climb cables relying solely on solar or laser power, as lifting this elevator car with a cable seems unlikely. The vehicle, equipped with heavy radiation shielding, would also be affected by the Coriolis force as it climbed, pulling the cable sideways and changing its angular momentum. To reduce this effect, the elevator car would have to maintain a careful speed, meaning a single trip up the elevator could take a week or more.

A Lunar Elevator?

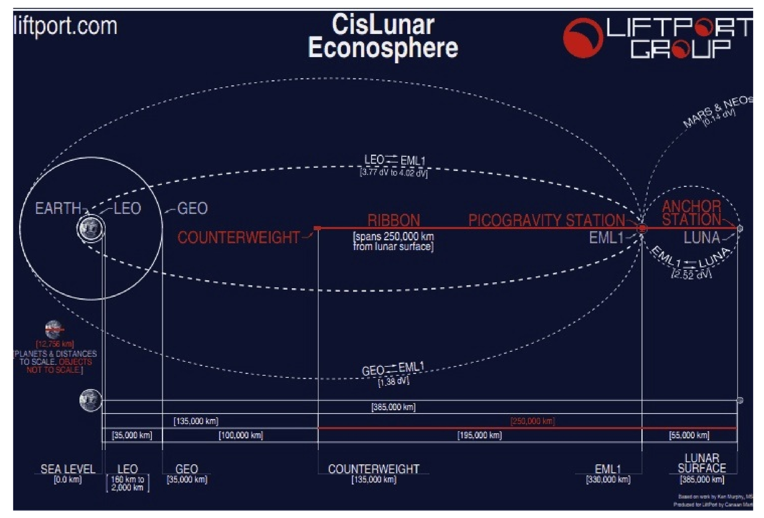


Figure 5: A proposed plan for a lunar elevator, by space company LiftPort.

Perhaps the best place to test the viability of the space elevator idea would be on our own moon. An elevator on the moon is already possible with current materials, as the stress on the cable would be only a fraction of that of a terrestrial space elevator. Applications for the lunar elevator would include sending valuable lunar regolith to Earth and enabling soft lunar landings. It may even be a necessary starting point in deploying the space elevator on Earth.

Several companies already have various ideas and propositions for the lunar elevator. Companies such as LiftPort and STAR, Inc. are optimistic, claiming that they can build an elevator that pays for itself in 19 payloads that will be active by 2025. Whether or not these claims come to fruition, a lunar prototype would be essential in proving the credibility - or perhaps unfeasibility - of the space elevator.

Data Tables

Gravitational constant	G	$6.67 \times 10^{-11} Nm^2kg^{-2}$
Earth's radius	R_E	$6370\ km$
Earth's angular velocity	ω	$7.272 \times 10^{-5} rad\ s^{-1}$
Earth's mass	M	$5.07 \times 10^{24}\ kg$
Geosynchronous orbit	R_G	$35,800\ km$ (above R_E)

Material	Density, kg/m^3	Max. tensile stress, GPa
Steel	7900	5.0
Kevlar	1440	3.6
Carbon Nanotube (theoretical)	1300	130